Dynamic generation of entangling wave packets in XY spin system with decaying long range couplings

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We study the dynamic generation of spin entanglement between two distant sites in an XY model with $1/r^2$-decay long range couplings. Due to the linear dispersion relation $\varepsilon(k) \sim |k|$ of magnons in such model, we show that a well-located spin state can be dynamically split into two moving entangled local wave packets without changing their shapes. Interestingly, when such two wave packets meet at the diametrically opposite site after the fast period $\tau = \pi$, the initial well-located state can be recurrent completely. Numerical calculation is performed to confirm the analytical result even the ring system of sizes N up to several thousands are considered. The truncation approximation for the coupling strengths is also studied. Numerical simulation shows that the above conclusions still hold even the range of the coupling strength is truncated at a relative shorter scale comparing to the size of the spin system.

INTRODUCTION

In quantum information processing, it is also crucial to generate entangled qubits, which can be used to perfectly transfer a quantum state over long distance. For optical system this task has been completed long time ago, but for a solid state systems it remains a great challenge both in experiment and theoretical setup to create quantum entanglement by a solid state device. Recently many proposals to entangle distant spins have been proposed based on various physical mechanisms [1, 2, 3, 4]. Most protocols for accomplishing quantum state transfer in a spin array base on the fixed inter-qubit couplings [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. The simplest coupled spin system with uniform nearest neighbor (NN) coupling has been studied by the pioneer work [6]. The paradigm to generate maximal entanglement and to perform perfect qubit-state transmission over arbitrary distance is the protocols using the pre-engineered inhomogeneous NN couplings [7, 14]. Among them, the practical scheme realizes the transmission of Gaussian wave packet as flying qubit via the spin system with uniform NN couplings [8, 16]. The advantages of such scheme rest with its fast transfer, which means that the period of transfer time is proportional to the distance, and the higher fidelity for longer distance. We also notice that there is a current interest in studying systems with long-range inter-qubit interactions [20, 21, 22].

In this paper, we revisit the issue of entanglement generation and quantum state transfer between two distant qubits in a qubit array with long-range inter-qubit interactions. An alternative way to construct a perfect medium for quantum state transfer may require the long-range interactions beyond NN interactions, but they should be required to decay rapidly in order to avoid the direct connecting coupling between two distant qubits that trivially causes quantum entanglement. Now we
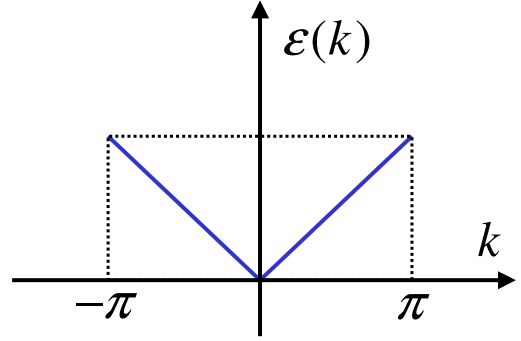

FIG. 1: *(color online) Schematic illustration for the ideal dispersion relation $\varepsilon(k) \sim |k|$ of a system which is shown to be a perfect entangler. Such kind of system can be realized by the XY model with pre-engineered long-range couplings.*

propose a novel protocol based on a pre-engineered XY-model with long-range $1/r^2$-decay interactions. It is found that such model has the same function as that of the modulated NN coupling spin model [7], offering significant advantages over other protocols in the tasks perfectly transferring quantum state and generating entanglement between two sites over longer distance.

This paper is organized as follows, in Sec. II, we consider a model with linear dispersion relation and show that it ensures the perfect state transfer and creation of entanglement between spatial separated qubits. In Sec. III we propose a spin model with $1/r^2$-decay interactions that process the desired dispersion relation. In Sec. IV we perform numerical calculation to confirm some of the obtained analytical results and the validity of the truncation approximation.
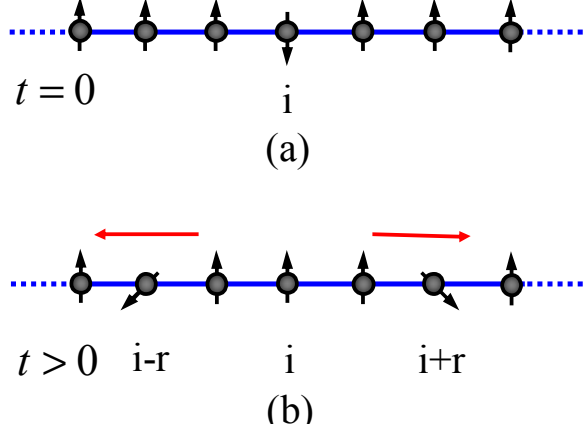
FIG. 2: *(color online) Schematic illustration of the generation process of two distant entangled qubits. At $t = 0$ the initial state is a single-spin flip on the saturated ferromagnet. At $t > 0$ the initial δ-pulse magnon is separated into two components which have maximal entanglement. At $t = \tau$, the recurrence time, the two sub-wave packets meet together and the initial state recurrent.*

FORMALISM

Pre-engineered model with linear dispersion relation

Usually, we regard a 1/2 spin as a qubit, and a coupled spin system as a qubit array respectively. The simple coupled qubit array is usually described by the spin-1/2 XY model. Our proposal makes the quantum spin array behave as a perfect spin-networks, in which a spin flip at any site (or the superposition of local single-spin-flip states) can evolve into two entangled, local wave packets. Consider the prescribed spin-1/2 XY-model on ring system with N sites. The Hamiltonian reads

$$H = 2 \sum_{i,r} J_r (S_i^x S_{i+r}^x + S_i^y S_{i+r}^y) \tag{1}$$

or

$$H = \sum_{i,r} J_r (S_i^+ S_{i+r}^- + S_i^- S_{i+r}^+), \tag{2}$$

where S_i^x, S_i^y and S_i^z are Pauli matrices for the site ith, and $S_i^\pm = S_i^x \pm i S_i^y$; J_r is the coupling strength for the two spins separated by the distance r. Since the Hamiltonian conserves spin, i.e., $[\sum_i S_i^z, H] = 0$, the dynamics can be reduced to that in some invariant subspaces. Thus we can only concentrate on the single excitation subspace hereafter. If a single-site flipped state can be correctly transferred, a qubit state should also be transferred correspondingly because the saturate ferromagnetic state with all spins up state $|0\rangle \equiv \prod_{i=1}^N |\uparrow\rangle_i$ is an eigenstate of the Hamiltonian. Actually, in the single-spin-flip subspace or

the single excitation (magnon) subspace, the basis states are denoted by the single-site flipped state (or δ-pulse) $|i\rangle = S_i^- |0\rangle$, $i \in [1, N]$. Therefore, if state $|i\rangle$ can evolve to state $|j\rangle$ after a period of time τ, $|j\rangle = \exp(-iH\tau) |i\rangle$, we have

$$(\alpha |\uparrow\rangle_j + \beta |\downarrow\rangle_j) \prod_{l \neq j}^N |\uparrow\rangle_l = e^{-iH\tau}(\alpha |\uparrow\rangle_i + \beta |\downarrow\rangle_i) \prod_{l \neq i}^N |\uparrow\rangle_l, \tag{3}$$

i.e., a qubit state $\alpha |\uparrow\rangle + \beta |\downarrow\rangle$ can be transferred from i to j. Furthermore, any single-magnon state can be expressed as

$$|\psi\rangle = \sum_i A_i S_i^- |0\rangle \equiv \sum_i A_i |i\rangle \tag{4}$$

and

$$|\psi\rangle = \sum_k D_k S_k^- |0\rangle \equiv \sum_k D_k |k\rangle \tag{5}$$

respectively in spatial $\{|i\rangle\}$ and momentum $\{|k\rangle\}$ spaces. Here, we have used the spin-wave operator

$$S_k^- = \frac{1}{\sqrt{N}} \sum_j e^{-ikj} S_j^-, \tag{6}$$

with discrete momentum $k = 2\pi n/N$, $n \in [-N/2, N/2 - 1]$.

We begin with assumption that there exists an optimal distribution of J_r, which ensures the single-magnon spectrum possessing a linear dispersion relation, i.e.

$$H_s = \sum_k \varepsilon_k |k\rangle \langle k| \tag{7}$$
$$\varepsilon_k = \frac{N}{2\pi} |k|$$

as illustrated in Fig. 1. In the following, we first show that such kind of systems can perform perfect state transfer and long range entanglement generation, and then we can provide a practical example that satisfies this linear dispersion relation.

Time evolution of wave packets in the linear dispersion regime

In order to investigate the dynamics of generating entanglement in the spin system with linear dispersion relation mentioned above, we concentrate on the case where the initial state is single-site flipped state $S_i^- |0\rangle$, $i \in [1, N]$. Intuitively, a well localized state has equal probability amplitudes with respect to momentum eigenstates $|\pm |k\rangle\rangle$, due to the inverse Fourier transformation

$$S_j^- = \frac{1}{\sqrt{N}} \sum_k e^{ikj} S_k^-$$

$$\xrightarrow{N \longrightarrow \infty} \frac{1}{\sqrt{N}} \frac{N}{2\pi} \int_{-\pi}^{\pi} e^{ikj} S_k^- dk$$

$$= \frac{\sqrt{N}}{2\pi} \int_0^{\pi} (e^{ikj} S_k^- + e^{-ikj} S_{-k}^-) dk. \qquad (8)$$

Thus it should be split into two classes of waves with $\pm |k|$ driven by the Hamiltonian with linear single-magnon dispersion relation (7). If the superposition of the two classes of waves are still well-localized (or form two wave packets) in real space, it will lead to the entanglement of the two wave packets.

Generally we first consider a wave packet dynamics with an initial state

$$|\psi(N_A, 0)\rangle = S_{N_A}^- |0\rangle = \frac{1}{\sqrt{N}} \sum_k e^{ikN_A} S_k^- |0\rangle, \qquad (9)$$

which is a single flip at site N_A. The time evolution starting with this state can be calculated as

$$\begin{aligned} |\psi(N_A, t)\rangle &= e^{-iHt} |\psi(N_A, 0)\rangle \\ &= |\phi_-(N_A, t)\rangle + |\phi_+(N_A, t)\rangle \end{aligned} \qquad (10)$$

where

$$|\phi_\pm(N_A, t)\rangle = \mp \frac{\sqrt{N}}{2\pi} \int_0^{\mp\pi} e^{ik(N_A \pm \frac{N}{2\pi} t)} dk |k\rangle \qquad (11)$$

denote the two wave packets with explicit forms in the real space

$$\begin{aligned} |\phi_\pm(N_A, t)\rangle &= \mp \frac{\sqrt{N}}{2\pi} \int_0^{\mp\pi} e^{ik(N_A \pm \frac{N}{2\pi} t)} dk \frac{1}{\sqrt{N}} \sum_j e^{-ikj} |j\rangle \\ &= \frac{1}{2\pi} \sum_j \left[\frac{\mp e^{\mp i\pi(N_A - j \pm \frac{N}{2\pi} t)} \pm 1}{i\left(N_A - j \pm \frac{N}{2\pi} t\right)} \right] |j\rangle \end{aligned} \qquad (12)$$

We will show that such two wave packets are localized and nonspreading, with velocity $v_\pm = \pm N/2\pi$, respectively. Actually, from Eq. (12) one have

$$|\phi_\pm(N_A, t + t_0)\rangle = \left| \phi_\pm(N_A \pm \frac{N}{2\pi} t, t_0) \right\rangle \qquad (13)$$

or

$$e^{-iHt} |\phi_\pm(N_A, t_0)\rangle = T_{\frac{N}{2\pi} t} |\phi_\pm(N_A, t_0)\rangle \qquad (14)$$

where T_a acts as translational operator for the states $|\phi_\pm(l, t)\rangle$: $T_a |\phi_\pm(l, t)\rangle \equiv |\phi_\pm(l \pm a, t)\rangle$. In general, the spatial coordinate is treated as discrete variable while the temporal coordinate is continuous. In order to make T_a to be operable, the translational spacing a should be integer. In the following, we only consider the states at discrete instant t, which ensures a to be integer, i.e., $Nt/2\pi = [Nt/2\pi]$ (the integer part of $Nt/2\pi$). In terms of

$$l_\pm(t) = N_A - j \pm \left[\frac{N}{2\pi} t \right], \qquad (15)$$

the wave packets (12) can be re-written as

$$|\phi_\pm(N_A, t)\rangle = \frac{1}{2\pi} \sum_j \left\{ \frac{1}{il_\pm(t)} [\mp e^{\mp i\pi l_\pm(t)} \pm 1] \right\} |j\rangle. \quad (16)$$

Obviously, Eq. (14) shows that state $|\phi_\pm(N_A, t)\rangle$ is shape-invariant with velocity $v_\pm = \pm N/2\pi$. Now we show the locality of these states. The projections

$$\langle j | \phi_\pm(N_A, t)\rangle = \begin{cases} \frac{1}{2}, & l_\pm(t) = 0 \\ \mp \frac{i}{\pi l_\pm(t)}, & \text{odd } l_\pm(j, t) \\ 0, & \text{even } l_\pm(t) \neq 0 \end{cases} \qquad (17)$$

of $|\phi_\pm(N_A, t)\rangle$ onto the localized state $|j\rangle$ shows that the shape-invariant states are well-localized. And each wave packet has the probability amplitude

$$|\phi_\pm(N_A, t)|^2 \simeq \frac{1}{4} + \frac{2}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} = \frac{1}{2}, \qquad (18)$$

which indicates that the initial state (9) is splited into two wave packets completely. On the other hand, the dispersion (7) requires the translational invariance of the Hamiltonian since the momentum is the conserved quantity, which leads to the periodicity of the states $|\phi_\pm\rangle$

$$\left| \phi_\pm(N_A \pm \frac{N}{2\pi} t, t)\right\rangle = \left| \phi_\pm(N_A \pm \frac{N}{2\pi} t \mp m_\pm N, t)\right\rangle, \qquad (19)$$

where $m_\pm = 1, 2, \cdots$, are integers. Obviously, when $\tau = (m_+ + m_-)\pi$, the initial state $|\psi(N_A, 0)\rangle = S_{N_A}^- |0\rangle$ evolves to

$$|\psi(N_A, \tau)\rangle = S_{N_A - \frac{N}{2}(m_+ - m_-)}^- |0\rangle, \qquad (20)$$

which is just the recurrence of $|\psi(N_A, 0)\rangle$ on the positions $N_A - N(m_+ - m_-)/2$ at instants $(m_+ + m_-)\pi$. The physics of this phenomenon can be understood as the interference of two wave packets (12). It also accords with the prediction for the system with spectrum-symmetry matching condition (SSMC) [14, 18].

In the single magnon invariant subspace, the single flip states $\{|j\rangle\}$ at site j constitute the complete basis. Then the above conclusion can be applied to any states in this subspace. Consider an arbitrary initial state $|\Phi(0)\rangle = \sum_j A_j |j\rangle = \sum_j A_j |\psi(j, 0)\rangle$, which is a coherent superposition of single flip states. At time t, it evolves to

$$|\Phi(t)\rangle = e^{-iHt} |\Phi(0)\rangle = |\Phi_+(t)\rangle + |\Phi_-(t)\rangle, \qquad (21)$$

where

$$|\Phi_\pm(t)\rangle = \sum_j A_j |\phi_\pm(j,t)\rangle \qquad (22)$$

represent two invariant-shape states. Accordingly, at instants $\tau = (m_+ + m_-)\pi$, the final state is a translation of $|\Phi(0)\rangle$

$$|\Phi(\tau)\rangle = T_{\frac{N}{2}(m_+ - m_-)} |\Phi(0)\rangle. \qquad (23)$$

Furthermore, during the period of time $t \neq \tau$, wave packets $|\Phi_\pm(t)\rangle$ are still well localized in space if the initial state $|\Phi(0)\rangle$ is local. Then the revival of a wave packet can be used to implement perfect quantum state transfer.

Entanglement of two separated spins

Now we turn our attention on the entanglement of the two separated spins induced by the wave packets (12). The reduced density matrix of a state $|\Phi(t)\rangle$ for two spins located at sites i and j [23, 24] has the form

$$\rho_{ij} = \begin{pmatrix} v_{ij}^+ & 0 & 0 & 0 \\ 0 & w_{ij} & z_{ij} & 0 \\ 0 & z_{ij} & w_{ij} & 0 \\ 0 & 0 & 0 & v_{ij}^- \end{pmatrix} \qquad (24)$$

with respect to the standard basis vectors $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, and $|\downarrow\downarrow\rangle$. Here, the matrix elements are

$$\begin{aligned} v_{ij}^\pm &= \frac{1}{4} + \frac{1}{2} \langle\Phi(t)| [2S_i^z S_j^z \pm (S_i^z + S_j^z)] |\Phi(t)\rangle, \\ w_{ij} &= \frac{1}{4} - \langle\Phi(t)| S_i^z S_j^z |\Phi(t)\rangle, \\ z_{ij} &= \frac{1}{2} \langle\Phi(t)| (S_i^+ S_j^- + S_i^- S_j^+) |\Phi(t)\rangle. \end{aligned} \qquad (25)$$

Correspondingly, the concurrence of two spins located at sites i and j for a state $|\Phi(t)\rangle$ can be calculated by

$$C_{ij} = \max\left\{0, 2\left(|z_{ij}| - \sqrt{v_{ij}^+ v_{ij}^-}\right)\right\}, \qquad (26)$$

Since the state we concern is in the invariant subspace with $S_z = N/2 - 1$, we have $v_{ij}^+ = 0$ and then the concurrence reduces to

$$C_{ij} = \left|\langle\Phi(t)| (S_i^+ S_j^- + S_i^- S_j^+) |\Phi(t)\rangle\right|. \qquad (27)$$

Consider an initial state having reflectional symmetry with respect to a point N_A, which has the form

$$|\Phi(0)\rangle = f_{N_A}(0) |N_A\rangle + \sum_{j=1}^{N/2} f_j(0)[|N_A + j\rangle + (-1)^R |N_A - j\rangle], \qquad (28)$$

where R=even (odd) represents the parity of the state under the reflection. The concurrence between two spins at $N_A + j$ and $N_A - j$ for the state $|\Phi(t)\rangle$ is

$$\begin{aligned} C(j,t) &= C_{N_A+j, N_A-j} \\ &= \left|\langle\Phi(t)| (S_{N_A+j}^+ S_{N_A-j}^- + S_{N_A+j}^- S_{N_A-j}^+) |\Phi(t)\rangle\right| \\ &= 2\left|(-1)^R f_j^*(t) f_j(t)\right| \\ &= 2\left|f_j(t)\right|^2. \end{aligned} \qquad (29)$$

Obviously, the total concurrence $\sum_{j>0} C(j,t) = 1$ due to the normality of the wave function $|\Phi(t)\rangle$. Usually, the total concurrence is regarded as the concurrence of two wave packets. Taking the single flip at N_A in the form (9) to be the initial state as an example, straightforward calculation shows that non-zero concurrences are

$$\begin{aligned} C(j,t_0) &= \frac{1}{2} \\ C(j,t_n) &= \frac{2}{(2n-1)^2\pi^2}. \end{aligned} \qquad (30)$$

For this to occur, one requires that $t_0 = 2\pi j/N$, $t_n = 2\pi[j + (2n-1)]/N$, $n = 1,2,....$ It indicates that the concurrence with magnitudes (30) for two spins at N_A+j and $N_A - j$ can be generated at the moment t_0, t_1, t_2, ..., t_n.

MICROSCOPIC LONG-RANGE INTERACTION MODEL

In this section, we consider the possibility to realize the above scheme in a system which possesses the linear dispersion relation based on an one-dimensional arrangement of spins (qubits) coupled by long-range interactions. Actually, using the identity

$$|2\xi| = \frac{N}{2} - \sum_{r=odd}^{\infty} \frac{4N}{r^2\pi^2} \cos\frac{r\pi(2\xi)}{N}, \qquad (31)$$

we have

$$\frac{N}{2\pi} |k| \approx J_0 + \sum_{r=odd}^{N/2-1} 2J_r \cos(kr), \qquad (32)$$

where

$$J_0 = \frac{N}{4}, \quad J_r = \frac{N}{r^2\pi^2}. \qquad (33)$$

Then the Hamiltonian matches the case (7) in single-

magnon invariant subspace can be rewritten as

$$
\begin{aligned}
H &= h_0 + \sum_{r=odd}^{N/2-1} J_r h_r, \\
h_0 &= N J_0, \\
h_r &= \sum_i (S_i^+ S_{i+r}^- + h.c.).
\end{aligned} \tag{34}
$$

Notice that, although such model involves the long range interaction, J_r decays rapidly as r increases. So we call this as "microscopic long-range interactions". The above analytical analysis shows that such microscopic long-range interaction can lead to nontrivial long-range entanglement and QST.

NUMERICAL RESULTS AND TRUNCATION APPROXIMATION

In the previous sections, it is found that the microscopic long-range interaction can lead to nontrivial long-range entanglement and QST for large N limit system. In this section, numerical simulations are performed for finite systems to illustrate the result we obtained above and investigate the application of it. The numerical exact diagonalization method is employed to calculate the time evolution of the single flip state (9) and the Gaussian wavepacket in the following form

$$
|\psi_G(N_A, \alpha)\rangle = \frac{1}{\sqrt{\Omega}} \sum_i e^{-\frac{\alpha^2}{2}(N_A - i)^2} S_i^- |0\rangle \tag{35}
$$

in the finite N system. Here, Ω is the normalized factor and N_A, α determine the center and the shape of wavepacket. In Fig. 3 and 4, the time evolution of a single flip state (9) and a Gaussian wave packet (35) of $\alpha = 0.1$ in a $N = 100$ ring system are plotted. It shows that the local initial states split into two wave packets in both cases and the wave packets keep their shapes without spreading. Similarly, numerical simulations are also employed for the open chain systems. The similar conclusions are also obtained for open chain system which will be discussed in the following.

A natural question is that what role the long-range coupling plays for the peculiar behavior of the propagation of the local wave in such a system. As shown above, the long-range hopping coupling decays rapidly. Then if the size of system is large enough, it is believed that the too long range coupling can be neglected. Thus, the "long range coupling" in the form (33) can be regarded as the relative local, or "microscopic long-range" coupling. In order to investigate this problem, or the boundary between the so called "long range" and "microscopic long-
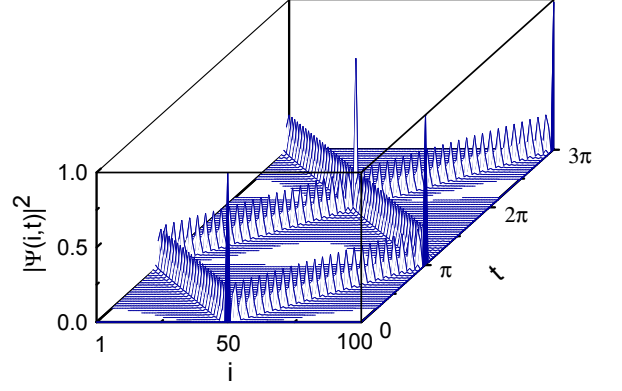

FIG. 3: (color online) Time evolution of a δ-pulse in a $N = 100$ ring system obtained by numerical simulation. It shows that the two sub-waves are local and keep the shape.
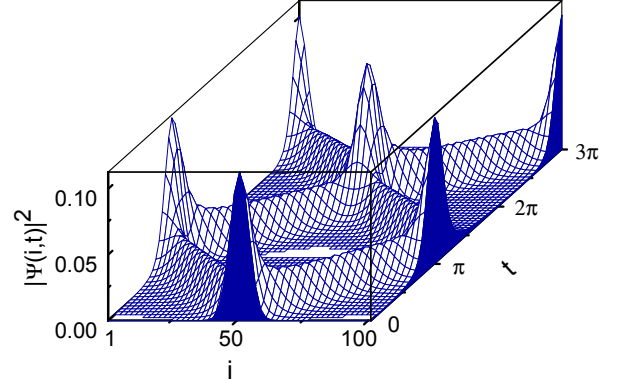

FIG. 4: (color online) Time evolution of a Gaussian wave packet of $\alpha = 0.1$ in a $N = 100$ ring system obtained by numerical simulation. It shows that the two sub-waves are local and keep the shape.

range" couplings, we consider the truncated Hamiltonian

$$
H = h_0 + \sum_{r=odd,\, <r_0} J_r h_r, \tag{36}
$$

where r_0 is the truncation distance. From the above analysis, the initial state $|\psi(N_A, 0)\rangle$ should recurrence at the positions N_A at instant 2π, if the system is perfect for the quantum state transfer. The autocorrelation $|A(t)| = |\langle \psi(N_A, 0) | \psi(N_A, t)\rangle|$ is appropriate quantity to investigate the role that r_0 plays. During the period of time $\sim [0, 3\pi]$, the maxima of autocorrelations $|A_{max}| = \max\{|A(t)|\}$ of the state (9) in the systems with $N = 500, 1000, 1500, 2000$, and $r_0 = 10, 20, ..., 100$ are calculated numerically by the exact diagonalization method. In Fig. 5, the dependence of the truncation distance r_0 on the quantity $|A_{max}|$ is plotted. It shows that $|A_{max}|$ approaches to 1 when r_0 is around 90, which is called the critical truncation distance or the boundary

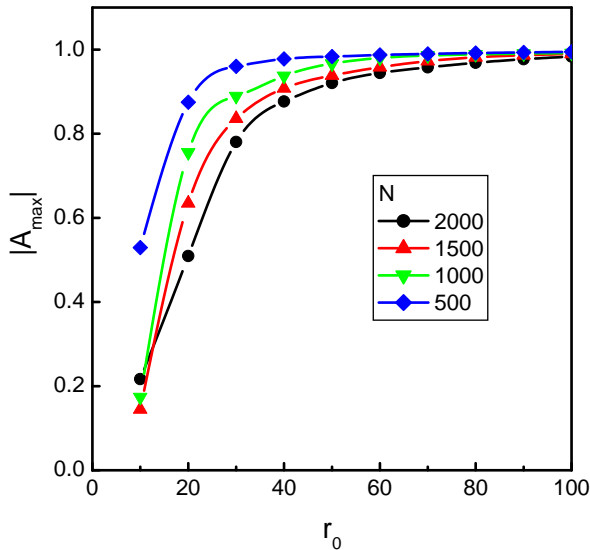FIG. 5: *(color online) The maximal autocorrelation functions of the initial δ-pulse state in the N-site systems with the truncation distance r_0. It demonstrates that critical r_0, at which $|A_{max}|$ start to approach to 1, do not depend on N strongly.*

FIG. 6: *(color online) The time-dependent concurrences of two spins separated by the distance $2l = 800$ in the chain system with $N = 1000$, $r_0 = 10, 15, 20, 30$ and 500. The results for $r_0 = 500$ is in agreement with the analytical analysis. The results for different truncation approximations show that range of LR coupling can be taken in a small scale due to the $1/r^2$-decay of coupling constants.*

between the "long range" and "microscopic long-range" couplings, for the systems with different N. It indicates that in the case of $N \gg r_0$, the wave packets still travel without spreading. By making use of this observation, we find that although the interactions between spins are "long range", the $1/r^2$-distribution of coupling strength allow us to limit the maximal interaction range while minimizing the degradation of the quantum coherence obtained for ideal model. Then we have the conclusion that the $1/r^2$-decay coupling can be regarded as local coupling, or microscopic long-range coupling. In other word, the robust long-range entanglement between two distant qubits is not due to the direct long range coupling interaction between them.

From the above observation, we find that, for large size system the long-range coupling ($r_0 \gtrsim 90$) can be neglected. Then in thermodynamic limit, a ring system is equivalent to a more practical system, open chain system. To demonstrate this, the numerical simulation is employed to investigate the concurrence $C(l, r_0, t)$ between two far separated sites $N/2 \pm l$ ($l \sim N/2$) for the N-site system with different truncation r_0. In Fig. 6, plots of $C(l, r_0, t)$ for the systems with $N = 1000, l = 400$ and different r_0 are presented. It shows that for an open chain system, long range entanglement between two distant qubits can be achieved via the time evolution of a single flip state. The maximal entanglement created by such system is 0.5, as measured by the two-point concurrence.
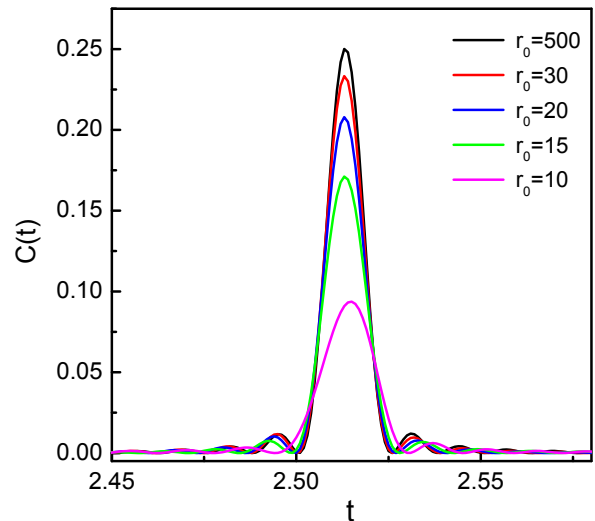
SUMMARY

In summary, the system with long-range coupling is investigated analytically and numerically. It is found that the $1/r^2$-decay long-range coupling model can exhibit approximately linear dispersion $\varepsilon \sim |k|$. The dynamics of such model possesses a novel feather that an initial local wave packet can be separated into two entangled local wave packets. Furthermore, during the traveling period each wave packets can keep their shapes without spreading. Numerical simulation indicates that there exists a critical truncation distance r_0, which limits the range of the interaction but not affects the generation of entanglement between two distant qubits in the distance $l \gg r_0$. This model open up the possibility to realize the solid-state based entangler for creating two entangled but spatially separated qubits.

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